

Print Name

Sign Name

MIDTERM EXAM 1

instructions:

This is a "closed book" exam; you should have no books, notes, calculators, other electronic devices, or scrap paper of your own available during the exam. Please use the exam booklet for all your work; you can use the backs of pages for scratch work.

For problem marked "Show work" you must show your work to get any credit, and partial credit is available.

For problems marked "Answer only", there's no part credit, and you don't need to show your work, just give the answer.

advice:

The exam has 100 points; the point-value of each problem is indicated.

Remember that it is OK to leave your answers as fractions, square roots, etc.

Scores

| #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | #10 | Tot | Letter |
|----|----|----|----|----|----|----|----|----|-----|-----|--------|
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| | | | | | | | | | | | |

1. (10 pts.) [Ans. only] Let $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(a) $\mathbf{a} \cdot \mathbf{b} =$

(b) $\mathbf{a} \times \mathbf{b} =$

(c) The length of the projection of \mathbf{a} upon $\mathbf{b} =$

(d) the unit vector that points in the same the direction as $\mathbf{a} =$

(e) Indicate for each whether it is a vector or a scalar (i.e. number), or does not make sense.

$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{j} \times \mathbf{k})$

$(\mathbf{a} \cdot \mathbf{a}) \times \mathbf{b}$

$\mathbf{a} \times (\mathbf{b} \cdot \mathbf{j})$

$(\mathbf{a} \times \mathbf{b}) + (\mathbf{j} + \mathbf{k})$

[show work]

2. (5 pts) Write an equation for the plane passing through the point (1,2,3) parallel to the plane $x - y + 4z = 5$.

3. (21 pts) [Ans. only]

DO 6 PICTURES AND 8 EQUATIONS.

Match each equation with its graph. The three equations are on this page, eight possible graphs are on the next page. Mark each equation with the number of its appropriate surface.

(Note: All the graphs are drawn with the x- and y- axes roughly horizontal and the z-axis pointing approximately "up". But the axes for the various graphs may have been rotated relative to those for other graphs, just to make the pictures easier to see. In particular, you should not assume that the x-axis and y-axis are the same from one picture to the other. Also the scales on the axes may not be the same within one picture, or between different pictures.)

a. $z = \sin(x) + \sin(y)$ _____

e. $x^2 + y + z^2 = 4$ _____
(read this carefully)

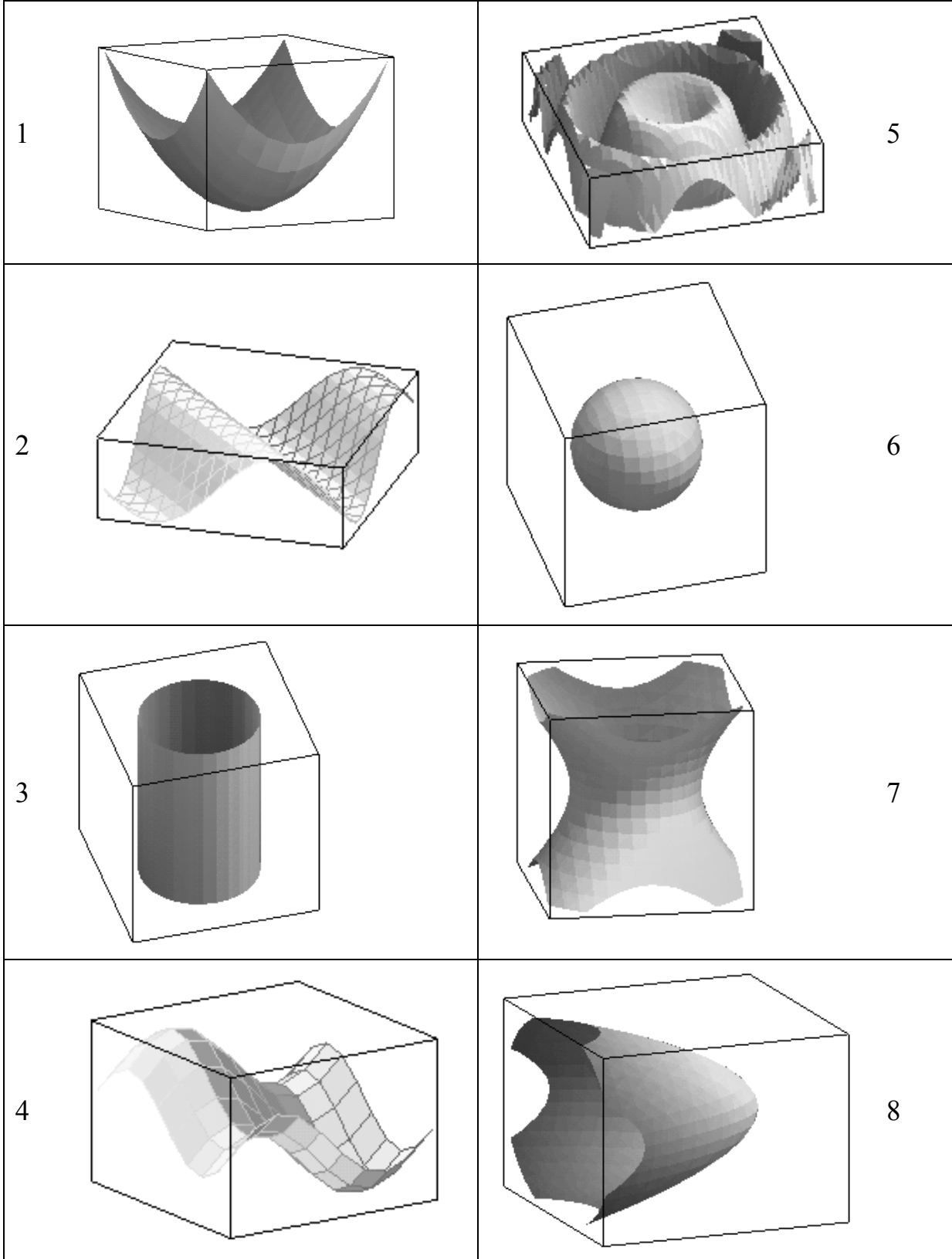
b. $z = \sin(x^2 + y^2)$ _____

f. $z = x^2 + y^2$ _____

c. $x^2 + y^2 - z^2 = 4$ _____

g. $x^2 + y^2 = 4$ _____

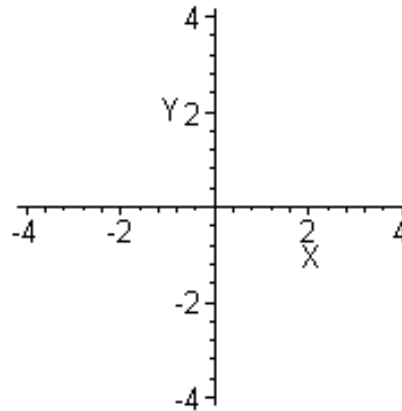
d. $z = \sin(x+y)$ _____



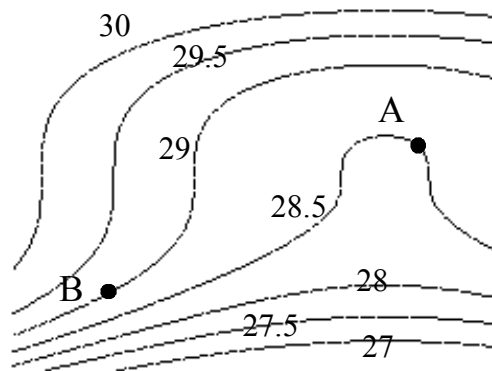
[show work]

4. (6 pts) Sketch the level curves corresponding to values $(1/2)$ and $(1/5)$ for the function (Draw the curves, with correct scale, on these axes.)

$$f(x,y) = \frac{1}{1 + (x^2 + y^2)}$$



5. (10 pts) [ans. only] Here is a contour diagram for the barometric pressure in some part of the country. Draw vectors representing the wind velocities at the points **A** and **B**. Pay attention to directions and [relative] magnitudes.



6. (6 pts) [show work] Here is a table of values for some linear. (i.e. "affine") function $z = f(x,y)$. Find a formula for the function $f(x,y)$.

| | | X | | | |
|---|-----|-----|-----|-----|-----|
| | | 5 | 5.1 | 5.2 | 5.3 |
| y | 2 | 9 | 9.2 | 9.4 | 9.6 |
| | 2.2 | 8.8 | 9 | 9.2 | 9.4 |
| | 2.4 | 8.6 | 8.8 | 9 | 9.2 |
| | 2.6 | 8.4 | 8.6 | 8.8 | 9 |

7 (24 pts) (answer only) Suppose $f(x) = x + xy^3$. Then....

(a) $\frac{\partial f}{\partial x} =$ _____.

(b) $\frac{\partial f}{\partial y} =$ _____.

(c) $\nabla f(5,2) =$ _____.

d) The directional derivative of f
in the direction of the vector $(3/5)\mathbf{i} - (4/5)\mathbf{j}$
at the point $(5,2)$ = _____.

e) The largest possible value for $f_{\mathbf{u}}(5,2)$,
where \mathbf{u} ranges over all unit vectors, = _____.

f) A direction vector (i.e. unit vector) \mathbf{w} for which $D_{\mathbf{w}}f(5,2) = 0$.

8. (6 pts) [show work] For a certain function $f(x,y,z)$, it is believed that $f(1,2,3) = 7$, $f_x(1,2,3) = 4$, $f_y(1,2,3) = 5$, and $f_z(1,2,3) = 6$.
Use this information to estimate $f(1.2, 1.9, 3.1)$.

9. (6 pts) [show work] Find an equation for the plane tangent to the surface $z = \sin(xy)$ at the point $(1, \pi/6, 1/2)$.

10. (6 pts) [ans. only]

Suppose x and y are given as functions of a parameter t , with

$$\begin{array}{l} x = t^2 + 1 \\ y = t^3 \end{array} \quad \text{and } f(x,y) \text{ is a function such that } \begin{array}{l} \frac{\partial f}{\partial x} = e^{(xy)} + xy e^{(xy)} \\ \frac{\partial f}{\partial y} = x^2 e^{(xy)} \end{array}$$

Viewing f as a function of t , calculate $\frac{df}{dt}$, **as follows**:

(a) State the version of the chainrule that is applicable here:

(b) Using your answer to (a), express $\frac{df}{dt}$ in terms of t alone
(but do not simplify it!)