Ph.D. Qualifying Exam in Analysis, by Paul Muhly and Lihe Wang

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The exam has two parts: real analysis and complex analysis. Each part has five problems and solve any four problems from five problems in each part.

If you want to try the fifth problems, the exam will be graded as the best four scores from the five.

Real Variables. Solve any four problems in these five problems from real variables.

1. Suppose $f_n(x), f(x)$ are measurable functions on $(0, 1)$. Suppose $f_n(x) \to f(x)$, it is true that $\int_{(0,1)} f_n(x)dx \to \int_{(0,1)} f(x)dx$? Give a proof or a counterexample.
2. Suppose f is a function defined $[0, 1]$ and suppose that $|f(x) - f(y)| \leq M|x - y|$ for all $x, y \in [0, 1]$ where M is a fixed constant. Prove that f is differentiable a.e. and $|f'(x)| \leq M$.
3. Show that there is no measurable set such that that $m(E \cap (a, b)) = \frac{b-a}{2}$ for all $a < b$, here $m(A)$ is the Lebesgue measure of A.
4. Suppose f_k, f are functions in $L^1([0, 1])$ such that $f_k(x) \to f(x)$, a.e., and that $\|f_k\|_{L^1} \to \|f\|_{L^1}$. Then $f_k \to f$ in L^1.
5. If $f \in L^1(R^1)$, show that $\sum_{n=-\infty}^{\infty} f(x + n)$ is convergent e.a to a function which has period 1.

Complex Variables. Solve any four problems in these five problems from complex variables.

1. Find all entire functions with the condition that $|f(z)| \leq A(1 + |z|^2)$ for some constant A.

2. Compute $\int_{\partial D(0,1)} (1 + \bar{z})^5 dz$.

3. Suppose f is holomorphic in the punctured disk $D(0, 1)\backslash\{0\}$. Suppose also that $|f| \leq \frac{1}{|z|^{0.5}}$. Prove f is differentiable at 0.

4. Suppose f is entire so that $Re(f) \geq 0$. Show f is a constant.

5. Suppose f is holomorphic in the unit disk. Prove that there exists z_n in the disk with $|z_n| \to 1$ that $f(z_n)$ is bounded.