Qualify exam 2019 Fall

2:30-5:20 pm, 60 SH, Aug 30, 2019

Notes:
i) Choose 4 questions of the following 5 questions to finish.

ii) In order to receive credit show all work.You may choose to solve the problems in a different order than listed below. You are not allowed to use calculators during the exam time.

(25 pts.) **Problem 1:** Consider the following dynamical system:

$$\begin{cases} x' = x^2 - y - 1 \\ y' = (x-2)y \end{cases} \tag{1}$$

in the (x,y)-plane.

a) Determine the nullclines of the system and find the fixed points

b) Compute the Jacobian matrix. Determine the linear stability of all fixed points

c) Draw the nullclines of the system.

(20 pts.) **Problem 2:** The flow of the system of differential equations

$$\begin{cases} x' = f(x,y) \\ y' = g(x,y) \end{cases} \tag{2}$$

is given by

$$\phi_t(x,y) = ((x + \frac{1}{5}y^3)e^{2t} - \frac{1}{5}y^3 e^{-3t}, ye^{-t}).$$

a) Determine the system, i.e., compute $f(x,y)$ and $g(x,y)$

b) Find the equilibria

c) Are there any periodic solutions

(20 pts.) **Problem 3:** Using Poincare-Bendixon theorem to show the the system

$$x' = x - y - x^3, \quad y' = x + y - y^3$$

has a periodic solution.

(20 pts.) **Problem 4:** Consider the dynamical system

$$\begin{cases} x' = ax + y + x^3 \\ y' = x - y \end{cases} \tag{3}$$

a) Find all fixed points of the system and give conditions on a for which the fixed points exist.

b) Use linear stability analysis to classify the fixed points you found in the previous question as functions of the parameter a.

c) Plot the bifurcation diagram for one of the components of the fixed points, for example x^*, against the parameter a. Label each branch plotted with the type of fixed point you found in the classification in question b). What kind of bifurcation(s) do you obtain?

(15 pts.) **Problem 5:** Consider the initial value problem

$$y' = -0.2(y - \sin(t)), y(\pi/4) = 1/\sqrt{2}$$

Use Euler method with step size $h = \pi/10$ to approximate the solution y$(\pi/4 + \pi/5)$.