Qualify exam 2020 Spring

2:30-5:20 pm, 212 PH, Jan 23, 2020

Notes:
i) Choose 4 questions of the following 5 questions to finish.

ii) In order to receive credit show all work.You may choose to solve the problems in a different order than listed below. You are not allowed to use calculators during the exam time.

(25 pts.) **Problem 1:** Consider the following dynamical system:

$$\begin{cases} x' = x^2 - y - 1 \\ y' = (x-2)y \end{cases} \tag{1}$$

in the (x,y)-plane.

a) Determine the nullclines of the system and find the fixed points

b) Compute the Jacobian matrix. Determine the linear stability of all fixed points

c) Draw the nullclines of the system.

(25 pts.) **Problem 2:** The flow of the system of differential equations

$$\begin{cases} x' = f(x,y) \\ y' = g(x,y) \end{cases} \tag{2}$$

is given by

$$\phi_t(x,y) = ((x + \frac{1}{5}y^3)e^{2t} - \frac{1}{5}y^3 e^{-3t}, ye^{-t}).$$

a) Determine the system, i.e., compute $f(x,y)$ and $g(x,y)$

b) Find the equilibria

c) Are there any periodic solutions

(25 pts.) **Problem 3:** Using Poincare-Bendixon theorem to show the the system

$$x' = -x - y + x(x^2 + 2y^2), \quad y' = x - y + y(x^2 + 2y^2)$$

has at least one closed orbit.

(25 pts.) **Problem 4:** Consider the nonlinear ODE system

$$\begin{cases} \dot{x} = -x \\ \dot{y} = y + x^2 \end{cases} \tag{3}$$

a) Determine the nonlinear flow $(\varphi_t)_{t\in\mathbf{R}}$ of (3).

b) Determine the flow $(\psi_t)_{t\in\mathbf{R}}$ of the linearization of (3) about the equilibrium point $(0,0)$.

c) Prove that $(\varphi_t)_{t\in\mathbf{R}}$ is topologically conjugate to its linearization $(\psi_t)_{t\in\mathbf{R}}$ about $(0,0)$, with conjugacy

$$h : \mathbf{R^2} \mapsto \mathbf{R^2}, \qquad \mathbf{h(x,y) = (x, y + \frac{1}{3}x^2)}$$

(25 pts.) **Problem 5:** Consider the initial value problem

$$y' = \frac{3}{x} + y, \quad y(1) = -1.$$

Use Euler method with step size $h = 0.4$ to approximate the solution $y(1.8)$.