Qualifying Exam: PDE, Fall, 2018

Choose any **Four** out of the five problems. Please indicate your choice.
Show all your work.

1. Show that if the C^1 initial data $f(x)$ has $f'(x_0) > 0$ for some x_0, then the C^1 solution of

$$u_t + (u(2-u))_x = 0, \quad x \in \mathbb{R}, t > 0, \quad u(x,0) = f(x)$$

must break down at some time $t > 0$.

2. (i) Solve the initial value problem

$$u_t - x^2 u_x = -u, \quad x \in \mathbb{R},\ t > 0,$$

$$u(x,0) = f(x), \quad x \in \mathbb{R}$$

where $f \in C^1(\mathbb{R})$.

(ii) Over which region in the x-t plane does the solution exist?

(iii) Write an upwind scheme for the above problem.

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3. Solve the following initial boundary value problem

$u_t - ku_{xx} = 0, \ \ x > 0, \ t > 0,$

$u(x, 0) = f(x), \ \ x \geq 0,$

$u_x(0, t) = 1, \ \ t \geq 0.$

where $k > 0$, $f \in C^2[0, +\infty)$, is bounded and $f'(0) = 1$.

4. (i) Solve the initial-boundary-value problem

$u_t = u_{xx}, \quad 0 < x < 1, \; t > 0,$

$u(0,t) = 1, \quad u(1,t) = 2, \quad t > 0,$

$u(x,0) = x^2 + 1, \quad 0 \le x \le 1.$

(ii) What is the limit of the solution as $t \to +\infty$?

5. Solve the initial value problem of the wave equation

$u_{tt} - 4u_{xx} = 0, \quad x \in \mathbb{R}, \ t > 0,$

$u(x,0) = -e^{-x^2}, \quad u_t(x,0) = 12xe^{-x^2}, \quad x \in \mathbb{R}.$