The University of Iowa
The College of Liberal Arts and Sciences
Spring, 2024

Title of Course: Functional Analysis II (MATH:7210:0001)
Course meeting time and place: 9:30am-10:20am /MWF, 105 MLH
Department of Mathematics: https://math.uiowa.edu/

Course ICON site: To access the course site, log into Iowa Courses Online (ICON) https://icon.uiowa.edu/index.shtml using your Hawk ID and password.

Course Home
The College of Liberal Arts and Sciences (CLAS) is the home of this course, and CLAS governs the policies and procedures for its courses. Graduate students, however, must adhere to the academic deadlines set by the Graduate College.

Instructor: Ionut Chifan (yo-nüts key-fun)
Office location: 1B MLH
Student drop-in hours: MWF 10:30pm-11:20pm, and by appointment.

Phone: 319-335-0777
E-mail: ionut-chifan@uiowa.edu
DEO Contact information: Professor Ryan Kinser, 14 MLH, ryan-kinser@uiowa.edu

Prerequisite: MATH: 5200,5210,6200

Description of Course
This will be a two-semesters course on the structural study of operator algebras (C*-algebras and von Neumann algebras) and their interactions with other related fields, especially group theory and ergodic theory. Specifically, course will explore depth a variety of applications and connections to noncommutative ergodic theory (orbit equivalence) and representation/geometric aspects in group theory (hyperbolicity, boundary techniques). The main purpose is to provide the fundamentals needed to start active research in these fields. After recalling the general theory of these objects, in the first part of the course, we will focus on the classification of C*-algebras and von Neumann algebras arising from discrete groups and their actions on probability spaces (so-called crossed products algebras). Within this framework we will first develop several fundamental tools both algebraic and analytic (and many with group theoretic/probability flavor) that are very useful to study various structural properties for von Neumann algebras including primeness, absence/uniqueness of Cartan subalgebras, solidity, strong solidity, etc. Then, using these we will prove a number of deep classification results for crossed product von Neumann algebras arising from large families of remarkable groups (eg hyperbolic groups, braid groups etc) and their natural actions (Bernoulli, compact) on probability spaces. In particular we will provide the first examples of discrete groups which

are entirely recognizable from theory reduced C* algebras and also their von Neumann algebras (such example emerged only in the last decade).
Along the way we will be discussing in depth several important topics in groups approximations (amenability, Haagerup property, weak amenability) and various weak forms of cohomology for group representations (biexactenss and more precisely construction of so-called quasi-cocycles and arrays on groups using methods in geometric group theory, etc). This will involve an in-depth study of groups acting on hyperbolic spaces.
In the second part the course will focus on applications of operator algebras to the study on noncommutative ergodic theory. After introducing the basic theory of stationary measures and Poisson boundaries and then we will derive several breaking applications to character rigidity, invariant random subgroups and Margulis normal subgroup theorem and Nevo-Furstenberg factor theorems. The course will culminate with a very recent result (by Peterson ’14 and Boutonnet-Houdayer ’19) confirming among other things a well-known conjecture by A. Connes (Fields Medalist). Additional topics will be covered if time permits.

Learning Objectives
This two semesters course is intended to introduce students to several new trends in functional analysis, specifically the classification of crossed product C*-algebras and von Neumann algebras associated with discrete groups and their actions on probability spaces. Our main goal is to present a self-contained approach towards the first examples of crossed product von Neumann algebras which completely ``remember'' their initial data. These results have been conjectured to exist for a long time (some say even from the beginning of the theory in early 40's) but the first concrete examples emerged only over the last 10-15 years ago. In addition to this the course will also introduce students to the first known examples of groups that are completely recognizable from their group von Neumann algebras and reduced groups C*-algebras. Finally, the course will focus on several applications of operator algebras to noncommutative ergodic theory, in particular the complete solution of a well-known open problem of a fields medalist. This is a rapid developing area of Mathematics with plenty of opportunities for graduate students to do active and rewarding research.

Textbook/Materials
For my lectures I will use the following online books and lecture notes as well as my personal notes that will be posted on ICON as the class progresses:

Claire Anantharaman and Sorin Popa, An introduction to II_1 factors
IIunV15.pdf (ucla.edu)

Nate Brown and Narutaka Ozawa: C*-algebras and finite-dimensional approximations, (2008), American Mathematical Society, Providence, Rhode Island.

The course covers the following topics in functional analysis, ergodic theory, group theory, C*-algebras, and von Neumann algebras using the textbooks and the above online lecture notes. (The first semester covered chapters 1-16 while the second will cover the remaining 17-23)

1.Operators on Hilbert spaces (definitions and basic properties, SOT and WOT topologies, compact operators and trace class operators)
2. Spectral theory of Banach algebras
3. Continuous and borelian functional calculus
4. GNS construction
5. von Neumann bicommutant theorem
6. Kaplanski continuity and density theorems
7. von Neumann algebras (predual, states space, geometry of projections and type classification, existence/uniqueness of the central trace, conditional expectation and basic construction)
8. Group-measure space von Neumann algebras (definitions, basic properties, examples, introduction to the classification program)
9. Amenable von Neumann algebras
10. Murray-von Neumann property Gamma and inner amenability
11. Generalities on representations theory of infinite groups
12. Groups with Kazhdan's property (T) (definitions, basic properties, lattices and other examples)
13. Positive definite and conditionally negative functions on groups (basic properties, examples, GNS construction, Schoenberg's Theorem)
14. Groups with Haagerup property (definitions, basic properties, examples)
15. Hyperbolic groups/graphs (definitions, basic properties, examples, crash course in hyperbolic geometry)
16. Acylindrically hyperbolic groups (definitions, basic properties, examples, group theoretic Dehn filling)
17. Biexact groups (examples, basic properties, connections with quasicohomology of group representations, construction of arrays and quasicocycles on groups using methods from geometric group theory)
18. Weak amenability for groups (definitions, basic properties, weak amenability of free/hyperbolic groups, non-examples)
19. Structural results for von Neumann algebras associated with biexact groups and their actions on probability spaces (primeness, solidity)
20. Classification of normalizers in von Neumann algebras arising from actions of biexact groups (after Ozawa-Popa '07, Popa-Vaes '2012)
21. Classification of group von Neumann algebras and reduced group C*-algebras; examples of W*-superrigid and C*-superrigid groups (a detailed account on analysis of commultiplication and height techniques for group operator algebras)
22. Margulis superrigidity theorem and Zimmer cocycle superrigidity theorem
23. Non-commutative measurable dynamics and applications to unitary representations (stationary measures and Poisson boundaries, induction and stationary actions, invariant random subgroups, Margulis Normal subgroup theorem, non-commutative Nevo-Zimmer theorem, Mautner property and singularity for G-actions, Peterson character rigidity theorem, classification of unitary representations of lattices in Lie groups, etc)

Academic Honesty and Misconduct
All students in CLAS courses are expected to abide by the CLAS Code of Academic Honesty. Undergraduate academic misconduct must be reported by instructors to CLAS according to these procedures. Graduate academic misconduct must be reported to the Graduate College according to Section F of the Graduate College Manual.

Student Collaboration:

The homework for this course is designed to help you master your knowledge related to the topics covered during lecture. As such, you may work on the homework problems and other assignments with others or use online resources. However, please be aware that to master the skills needed for this class, practice is required and that to do well on the final exam you will need to work many of these problems multiple times without help. Be sure to test your knowledge by doing much of the homework on your own. Even if you collaborate with your colleagues when solving the homework problems, I strongly encourage you not to copy, mot-a-mot, the solutions from others but instead try to write them in your own understanding. This is an excellent exercise proven to help students with their material comprehension.

Student Complaints
Students with a complaint about a grade or a related matter should first discuss the situation with the instructor and/or the course supervisor (if applicable), and finally with the Director or Chair of the school, department, or program offering the course.

Drop Deadline for this Course
You may drop an individual course before the deadline; after this deadline you will need collegiate approval. You can look up the drop deadline for this course here. When you drop a course, a “W” will appear on your transcript. The mark of “W” is a neutral mark that does not affect your GPA. Directions for adding or dropping a course and other registration changes can be found on the Registrar’s website. Undergraduate students can find policies on dropping and withdrawing here. Graduate students should adhere to the academic deadlines and policies set by the Graduate College.

Grading System and the Use of +/-
Final course grade will be assessed based on your performance in the following activities:

Final Presentation: 50% and Final Paper: 50%

As the class progresses, all grades will be recorded on ICON.

I will use the +/- grading system. Cutoffs for the letter grade are expected to follow the recommended scale given by CLAS below, and cutoffs for +/- are at the discretion of the instructor. You should not view this as a predetermined grade scale for assigning the final grade, but rather as a guaranteed minimum grading scale.

A [100,93); A- [90,93)

B+ [87,90); B [83,87); B- [80,83)

C+ [77,80); C [73,77); C- [70,73)

D+ [67,70); D [63,67); D- [40,63)

F [0, 40)

Next, I will briefly explain what this means. Let's say you finished the course with a 89%. This guarantees a B+. However, based on various factors such as material difficulty, the performance of the entire class in the course, etc, the grades may be curved in a way that the 89% corresponds to a grade of A-. But 89% will never be lower than a B+. In other words, any type of curving in this class is designed to only help you. If you are curious about your standing in the class, or your potential grade, please do not hesitate to reach out to me!

The grade of A+ will be awarded in extremely rare circumstances only for truly exceptional performance in the class.

Participation in class discussions: I strongly encourage you to actively participate in class discussions; ask questions or ask for more explanations whenever you feel confused; in this class there is NO stupid question!

Final Presentation: As the semester progresses the instructor will make available a number of topics from which the students can make their final presentations. Three weeks before the semester ends the students should meet with the instructor letting him know what and when they want to present.

Final Paper: Each student will prepare a final paper on a specific topic closely related to the material covered during the semester. Students will have to choose from 3-4 topics that will be made available by the instructor, 3 weeks before the end of the semester. Completing these papers will typically require additional readings and the instructor will provide new material and feedback as needed final submission. These final papers must be prepared in Latex (or other similar tool) and submitted electronically via email to the instructor. The due dates will be posted on ICON.

Date and Time of the Final Exam

There will be no final exam in this class

Calendar of Course Assignments and Exams

Week	Beg-End	No lectures	Chapters covered	Activities
1	1/16 - 1/19	2	8	

2	1/22 - 1/26	3	9	
3	1/29 - 2/2	2	10,11	
4	2/5 - 2/9	3	12	Hw
5	2/12 - 2/16	3	13	
6	2/19 - 2/23	3	15-16	
7	2/26 - 3/1	3	17	Hw
8	3/4 - 3/8	3	18	
9	3/18 - 3/22	3	19	
10	3/25 - 3/29	3	20	Hw
11	4/1 - 4/5	3	21	
12	4/8 - 4/12	3	22	Presentations
13	4/15 - 4/19	3	22	Presentations
14	4/22- 4/26	3	23	Presentations
15	4/29 - 5/3	3	23	Final paper

College of Liberal Arts and Sciences (CLAS) Course Policies
Attendance and Absences

Course attendance: Attendance is expected for each class meeting, as it will help you better understand the concepts covered in lectures. If you miss a class, you are responsible for any assignments/announcements made/material covered.
University regulations require that students be allowed to make up examinations which have been missed due to illness or other unavoidable circumstances (eg involvement in other UI authorized activities or sports, etc). So, students that missed an exam or assignment due to any of these reasons must notify the instructor immediately. They are also strongly encouraged to use the CLAS absence form on ICON under the Student Tools.
Students with mandatory religious obligations or UI authorized activities must discuss their absences with me as soon as possible. Religious obligations must be communicated within the first three weeks of classes.
Exam Policies

Communication: UI Email
Students are responsible for all official correspondences sent to their UI email address (uiowa.edu) and must use this address for any communication with instructors or staff in the UI community.

Other Expectations of Student Performance

Cell phones policy: I am expecting you to NOT use your cell phones, i-pads, or computers during the lecture time for other purposes than class related.

Changing grade policy: If I change your grade on a homework or exam you should always remind me in the same day by e-mail that I have changed your grade.

Where to Get Help

Students will find the following resources useful for this course:

Writing Center: http://www.uiowa.edu/~writingc/

Speaking Center: http://clas.uiowa.edu/rhetoric/for-students/speaking-center

Math Tutorial Lab: 125 MLH http://www.math.uiowa.edu/math-tutorial-lab

Tutor Iowa: https://tutor.uiowa.edu/

University Policies
Accommodations for Students with Disabilities
Basic Needs and Support for Students
Classroom Expectations
Exam Make-up Owing to Absence
Free Speech and Expression
Mental Health
Military Service Obligations
Non-discrimination
Religious Holy Days
Sexual Harassment/Misconduct and Supportive Measures
Sharing of Class Recordings